Electrical readout of a spin qubit without double occupancy.

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We identify a mechanism to read out a single solid-state electron spin using an all-electrical spin-to-charge conversion in a closed system. Our scheme uses three donors and two electron spins, one spin is the qubit, the other is a reference. The population in the third, originally ionized, donor is monitored with an electrometer. Energy dependent tunneling of the reference spin to the ionized donor is used to determine the state of the qubit. In contrast to previous methods [e.g. Kane, Nature (London), **393**, 133 (1998)] we avoid double electron occupancy of any site within the system, thereby eliminating the possibility of unwanted electron loss from the system. The single spin readout scheme described here is applicable to both electron and nuclear spin based quantum computer architectures.

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Understanding, observing and manipulating the quantum coherent properties of individual spins is an important endeavor for the physics community. Spin systems offer a superb probe of fundamental quantum properties. As such, they have been suggested and employed in various flavors as elements for quantum computers (QCs). It is believed by many that one of the best systems for realizing a scalable and practical spin-based QC is a solid-state system, that is fully compatible with existing technologies. This philosophy is encapsulated in several proposals, including the Kane proposal[1], which uses the nuclear spins of phosphorus in isotopically pure ^{28}Si as the qubits; the Loss and DiVincenzo[2] approach, where the qubits are electron spins in single-electron quantum dots; and the electron-spin-resonance approach of Vrijen *et al.*[3]. Progress towards realizing the Kane device has been recently reviewed[4].

Readout of a scalable spin-based QC relies on the ability to sense the state of single spins. The search for effective methods to measure single spins has involved researchers from many different disciplines. Numerous techniques have been suggested, including electrical spin-to-charge conversion[1,5], spin amplification using a paramagnetic dot[2], spin valves[2], magnetic-resonance-force microscopy[6], Raman transitions[7], far-infrared induced spin-to-charge conversion[8], optical readout[9] and the use of asymmetric confining potentials[10].

The original Kane proposal for spin to charge conversion requires two phosphorus donors: the qubit and a reference. First, the nuclear spin information is transferred to the electron spins. Then spin-dependent tunneling between the qubit and reference is used to determine whether the two spins are aligned parallel or anti-parallel. The tunneling creates a D^+D^- system, where the D^+ state is an ionized donor, and the D^- state a doubly occupied donor. The change in the charge distribution between the neutral and the D^+D^- system is monitored with a single electron transistor (SET)[11].

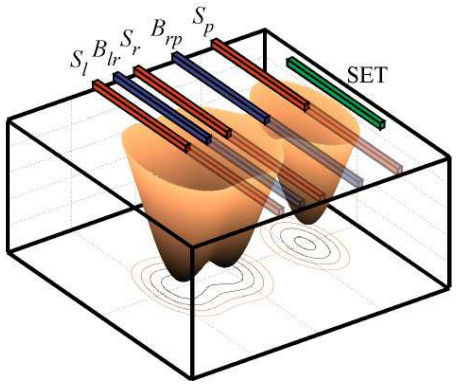

FIG. 1: Schematic showing triple well potential with top gates and readout SET. The two leftmost wells are strongly coupled, providing for a significant exchange interaction, the third well is further removed so as to act as a weakly coupled probe and is ionized prior to readout. The electron in the leftmost well is the spin qubit, l, the electron in the central well is the reference spin, r, and the third well is the probe site, p.

Although the D^+D^- state has been observed via far-infrared transmission[12], under the conditions required to adiabatically form the D^+D^- system in a top-gate controlled structure, it appears that the state will be quasi-bound, with a lifetime incompatible with SET readout[8]. It is therefore essential to determine alternative readout methods that avoid the D^- state problem. We present such an alternative here.

Our method is an all-electrical spin-to-charge conversion where an extra, unoccupied site (the probe site) is introduced to facilitate the readout. The arrangement is illustrated in Fig. 1. There are three potential wells, which could be derived from three donors, labelled l, r, and p, and two electrons. The electron in l is the qubit, the electron in r is the reference spin, and p is the probe

site. The energies of the states are controlled with shift gates, S_α ($\alpha = l, r, p$) and the tunneling and exchange interactions between sites are controlled with barrier gates B_{lr} and B_{rp}. The lr system is strongly coupled and the rp system weakly coupled, so as to probe the lr dynamics.

Readout is via a SET monitoring the population of the probe site. Our scheme discriminates between singlet and triplet states of the lr system which is equivalent to measuring the spin of l with known r[1]. We use the energy difference between the singlet and triplet states of the combined qubit and reference system to effect coherent tunneling into the probe, and hence qubit read out.

We illustrate our method for the Kane quantum computer, but the scheme is completely general and is applicable to most solid-state spin quantum computing schemes (including those of Refs.[2],[3] and[13]). More generally, this scheme provides an alternative tool for examining spin properties in quantum structures. Ionicioiu and Popescu[14] have proposed a different scheme using an ancilla charge state to read out a spin qubit.

Earlier, we presented a scheme for readout of a charge-based QC using a probe site[15] (three-site, one-electron case). Three-site two-electron models have been proposed for entangled current formation[16]. Such schemes differ qualitatively from that discussed here.

Given the added complexity of fabricating a triple-donor system over a more conventional two-donor system, and the increased coherence times that will be required, it is wise to identify regimes where our scheme is advantageous. As mentioned above, if the lifetime of the D^+D^- system is less than the SET readout time, two-donor spin to charge conversion will not be practical. Our scheme, combined with the charge shelving described below, circumvents such lifetime issues by avoiding the D^- state. Note that for spins in quantum dots[2] there are no problems due to double occupancy of the states, but there may be utility in the present proposal due to the lower required transfer potentials. We also require the following inequalities to be satisfied, $J/\hbar \gg \Omega_{rp} \sim 1/T_2^{rp}$ where J is the exchange interaction strength between electrons on sites l and r where we have assumed $J \gg J_{rp}, J_{lp}, \Omega_{ij}$ is the coherent tunneling rate on the ij transition with $\Omega_{lp} = 0$, and $1/T_2^{rp}$ is the (electrostatic) dephasing rate of charge motion on the rp transition. This inequality will become clearer below, however these inequalities are compatible with current thinking on lifetimes of spin and charge qubits.

We assume the SET functions as a weak measurement device, described by an effective T_2 time acting on the basis states of electron occupation of the probe donor. A full quantum treatment SET readout of a charge qubit has been performed by Wiseman *et al.*[17]. We further assume that this T_2 time is slow compared with the other timescales of the system, except for the T_1 time and is therefore not treated in our discussions. These assumptions are physically realistic for Si:P spin qubits, but more detailed calculations must be performed to quantitatively determine the dynamics.

Detailed analyses of the effects of top gates on the spin states of coupled two-site, two electron systems have been performed[18,19]. These treatments perform calculations to derive couplings in realistic systems. We make no attempt to replicate these important results, rather we *assume* the existence of appropriate interactions to illustrate the concepts of our scheme.

To understand the mechanism for transfer, we analyze the Hamiltonian for the two-electron, three-site problem on the basis of states $|\alpha\beta\bullet\rangle$, $|\alpha\bullet\beta\rangle$, $|\bullet\alpha\beta\rangle$ for $\alpha, \beta = \downarrow, \uparrow$ where the ordering is l, r, p and $\bullet$ denotes an unoccupied site.

$$\begin{aligned}\mathcal{H} &= \sum_{\alpha=\downarrow,\uparrow}\Bigg[\sum_{i=l,r,p} E_i b^\dagger_{i,\alpha} b_{i,\alpha} \\ &+ \hbar\Omega_{lr}\left(b^\dagger_{l,\alpha} b_{r,\alpha} + h.c.\right) + \hbar\Omega_{rp}\left(b^\dagger_{r,\alpha} b_{p,\alpha} + h.c.\right)\Bigg] \\ &+ 4J\sum_{i,j=l,r} S_i \cdot S_j + g\mu_B B \sum_{i=l,r,p} S^z_i, \qquad (1)\end{aligned}$$

where $b_{i\alpha}$ is the annihilation operator for an electron on site i with spin α; B is the magnetic field; E_i is the electrostatic energy of an electron on site i; $S_i = b^\dagger_{i\downarrow} b_{i\uparrow}$; $S^z_i = (1/2)\left(b^\dagger_{i\uparrow} b_{i\uparrow} - b^\dagger_{i\downarrow} b_{i\downarrow}\right)$ and we define $B^* = g\mu_B B$ as the Zeeman energy splitting.

The eigenvalues of the system as a function of E_p are illustrated in Fig. 2 (a) where we have chosen $B^* = J/5$ and $\Omega_{lr} = 5J$, so the singlet-triplet sub-manifolds are well resolved and the symmetric and anti-symmetric manifolds with p occupied $[(|\alpha\bullet\beta\rangle \pm |\bullet\alpha\beta\rangle)/\sqrt{2}$, $\alpha, \beta = \uparrow, \downarrow]$ similarly well resolved. Anti-crossings in the evolution of the eigenvalues indicate where the states change their character. If the system is initially prepared with electrons in sites l and r, then an anti-crossing for adiabatically swept E_p corresponds to electronic transfer from r to p. There are two sets of diagonal lines, corresponding to the final state of the electron in the lr system being in either the symmetric (lower energies) or anti-symmetric (higher energy) superpositions. Within each set, there are two biases where charge transfer anti-crossings occur, which correspond to transfer of an electron from the singlet state at $E_p = \pm\Omega_{lr} - 3J$, and from the triplet states at $E_p = \pm\Omega_{lr} J$. Note that all three triplet states within the same charge symmetry sub-manifold anti-cross at the same bias, suggesting that this scheme is not able to resolve the individual triplet components.

To determine the expected readout, we take the trace of the density matrix, ρ, over all states with a population in the probe dot p, which we term $\mathcal{S}$. We perform a transient analysis of $\mathcal{S}$ by solving the density matrix equations of motion, $\dot{\rho} = (-i/\hbar)[\mathcal{H}, \rho]$, for various initial conditions as a function of probe bias.

Results showing $\mathcal{S}$ as a function of bias and time are presented in Fig. 2 (b) - (d). The figures show coherent oscillations in $\mathcal{S}$, peaking at the resonance bias, with the oscillation frequency increasing away from resonance. Fig. 2 (b) shows the bias spectroscopy when the system

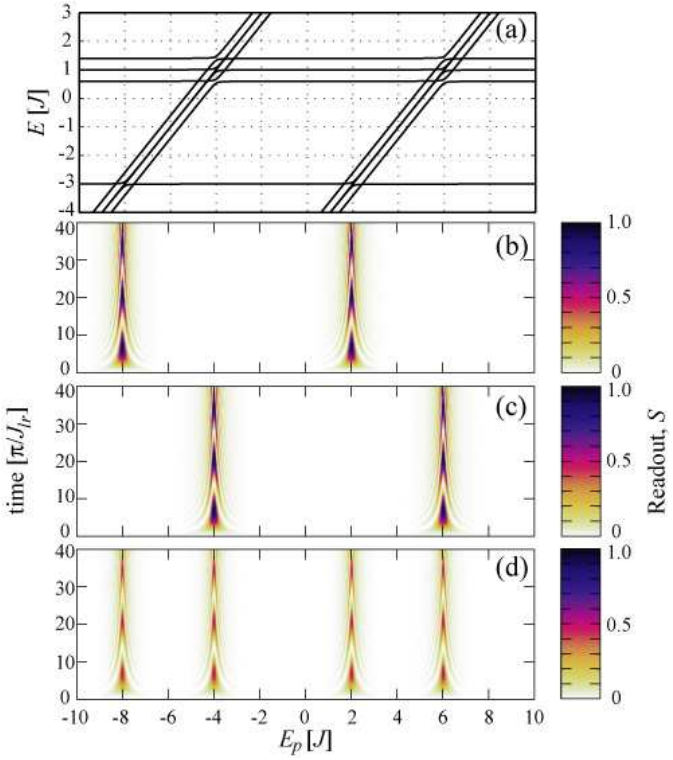

FIG. 2: (a) Eigenvalues for the two-spin, three-well case as a function of the energy of the probe state, E_p, for constant Zeeman splitting and exchange interaction. The states with occupancy in the third site migrate upwards in the figure. Anti-crossings at biases corresponding to resonance with the singlet and triplet states signify charge transfer to the third well. All triplet anti-crossings appear at the same E_p, indicating one cannot resolve the individual triplet states. (b) - (d) Transient bias spectroscopy showing readout variable $\mathcal{S}$ as a function of E_c/J and time (in units of $\pi\hbar/J$ for $B^*/J = 0.4$, $\Omega_{rp}/J = 0.1$, $E_l = E_r = 0$ and various initial conditions, (b) any of the triplet states, (c) singlet state, and (d) $\rho(0) = |\uparrow\downarrow\bullet\rangle$, which is a superposition of singlet and one of the triplet states.

is initialized in the singlet state, $(1/\sqrt{2})(|\uparrow\downarrow\bullet\rangle - |\downarrow\uparrow\bullet\rangle)$. Population is transferred to the probe when $E_p = \pm\Omega_{lr} - 3J$. Fig. 2 (c) shows $\mathcal{S}$ for the system initialized in either of the triplet states, $|\uparrow\uparrow\bullet\rangle$, $(1/\sqrt{2})(|\uparrow\downarrow\bullet\rangle + |\downarrow\uparrow\bullet\rangle)$ and $|\downarrow\downarrow\bullet\rangle$. This state has a different spectroscopic signature, with readout observed at $E_p = \pm\Omega_{lr} + J$, with the individual triplet states are unresolvable. In Fig. 2 (d) we present results obtained when the initial state was $|\uparrow\downarrow\bullet\rangle$. This state corresponds to a superposition of the singlet state and the symmetric state, i.e. $|\uparrow\downarrow\bullet\rangle = (1/\sqrt{2})\left[(|\uparrow\downarrow\bullet\rangle + |\downarrow\uparrow\bullet\rangle) + (|\uparrow\downarrow\bullet\rangle - |\downarrow\uparrow\bullet\rangle)\right]$. Because of this superposition, we observe two sets of biases (one per submanifold) where charge transfer to the probe donor is observed, and this is clearly seen in the spectroscopic signature presented in Fig. 2 (d). Unlike the charge-qubit readout[15] and optical case[20], there is no interference between the features in Fig. 2 (d). This is because there are no shared final states in the spin readout scheme, and therefore no interference. Inclusion of a measurement induced T_2 here will tend to wash out the oscillations in all cases shown in Fig. 2, allowing $\mathcal{S}$ to evolve to a steady state in a time commensurate with T_2.

For single-shot readout for a QC, we propose implementing a form of adiabatic fast passage (AFP)[21] on the rp transition. This is analogous to an earlier suggestion for single-shot readout of a charge-qubit in the superposition basis[15]. The advantages of AFP over bias spectroscopy include insensitivity to coherent oscillations on the $r-p$ transition and robustness to gate errors. Although we do not discuss decoherence in this work, it is important to realize that the minimum length of time to implement an AFP gate sweep will be of order $10\pi\hbar/J$. Thus the decoherence time should be long compared to this timescale. Given the already demanding requirements for dephasing in QCs[22], i.e. that the decoherence time should be $10^3 - 10^6$ times the coherent oscillation time, $\hbar/J$, then if we *assume* that construction of a scalable qubit is possible, the added overhead of implementing the AFP sweep is negligible.

To effect the AFP gate sweep, we vary E_p and Ω_{rp} according to

$$\begin{aligned} E_p &= \Omega_{lr} + 2J_{lr}\left(1 - t/t_{\max}\right), \\ \Omega_{rp} &= \Omega_{rp}^{\max}\left[1 - \cos\left(2\pi t/t_{\max}\right)\right]/2, \end{aligned} \tag{2}$$

where $\Omega_{rp}^{\max} = 0.3J_{lr}/\hbar$, $t_{\min} = 0$, $\Omega_{lr} = 10J_{lr}/\hbar$ and $t_{\max} = 10\pi\hbar/J_{lr}$. To illustrate this, Fig. 3 (a) shows $\Omega_{rp}(t)$ (left axis) and E_p (right axis). Note that in keeping with conventional AFP schemes, the scheme is fairly insensitive to the exact form of Ω_{rp}. $\mathcal{S}$ as a function of time is presented in 3 (b) for the three cases of the $l-r$ system being initially in the singlet state (solid line), triplet state (long dashes) and superposition state $|\uparrow\downarrow\bullet\rangle$ (short dashes).

With the addition of gate noise and decoherence, nonadiabatic techniques would only be expected to transfer on average half an electron to the probe site. By contrast, the AFP scheme will transfer a full electron (to arbitrary precision) even in the presence of gate errors. Therefore this scheme is compatible with single-shot readout, whereas the nonadiabatic scheme is not, yielding only a statistical result.

Until now we have deliberately concentrated on an arbitrary system to highlight the generality of our readout mechanism. We conclude by turning our attention specifically to readout of a Kane-type QC[1]. For lr donor spacing of ~ 15nm, the potential on a B gate required to shift the exchange coupling from $J = 0$ to $J \sim 0.1$meV is 1V[19]. At these separations, we would expect $\Omega_{lr} \sim 1$meV. To achieve the bias sweep necessary our AFP protocol, we would need to vary E_p smoothly from $E_p = 1.2$meV to $E_p = 1.0$meV. TCAD Modelling[23] suggests that an S gate to the right of the donor for a charge qubit will shift the potential by $\sim$ 4meV for a change in gate potential of 1V. This implies that the S gate potential must be controlled to of order tens of mV, which is achievable using conventional technology. The requirement for

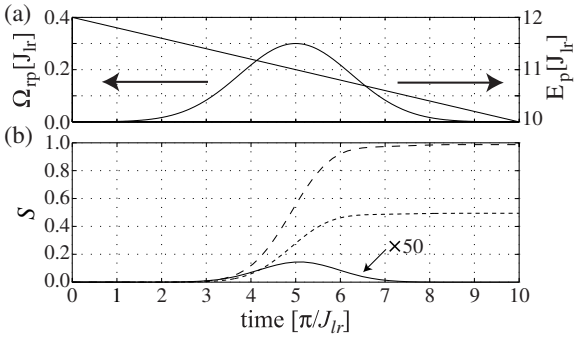

FIG. 3: Adiabatic fast passage (AFP) trajectories and readout for various initial conditions. (a) shows Ω_{rp} (left axis) and E_p (right axis) as a function of time for the AFP sequence. (b) shows $\mathcal{S}$ as a function of time for the $l-r$ system in the singlet state (solid line), triplet state (long dashes) and a superposition of singlet and triplet states (short dashes). The singlet readout has been multiplied by 50 to be visible on this scale. $\mathcal{S}$ migrates smoothly to the required value suggesting this is an appropriate mechanism for performing readout.

$\Omega_{rp}^{\max} \sim 0.3 J_{lr}$ would imply a spacing between r and p of around 25nm, again achievable with current technology. Other QC schemes will have quite different site-gate couplings due to different geometries. One would normally expect larger couplings for schemes where the quantum sites are extended structures (e.g. GaAs quantum dots, where quantum coherence has been shown[24]) than for the single donors envisaged here.

In summary, we have presented a high-fidelity, single-shot scheme for performing readout of a spin qubit, making use of a reference qubit and an empty probe site. The energy difference between the singlet and triplet states of the spin-reference system is probed using bias spectroscopy to the probe site, and the change in charge on the probe is monitored with a SET. This constitutes a form of spin-to-charge conversion and charge shelving, where the spin information is transferred to the charge of a long-lived probe site. This enables the use of a measurement device where the measurement time is longer than the coherence time of the qubit. Our techniques should be applicable to a wide range of different spin-based quantum computing schemes.

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